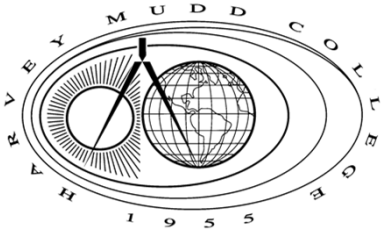
Introduction to CMOS VLSI Design

Lecture 12: Datapath Functional Units

David Harris



Harvey Mudd College

Spring 2004

Outline

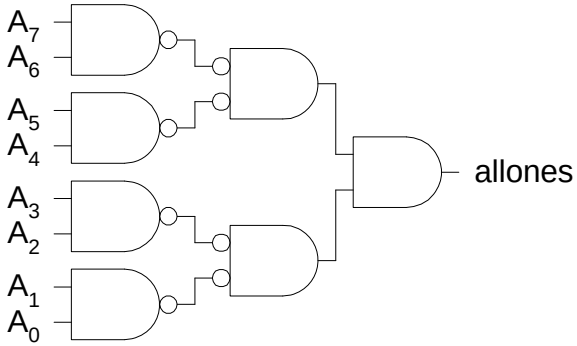
- Comparators
- Shifters
- Multi-input Adders
- Multipliers

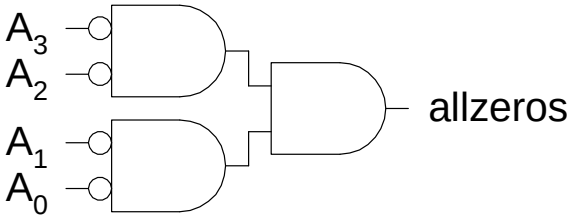
Comparators

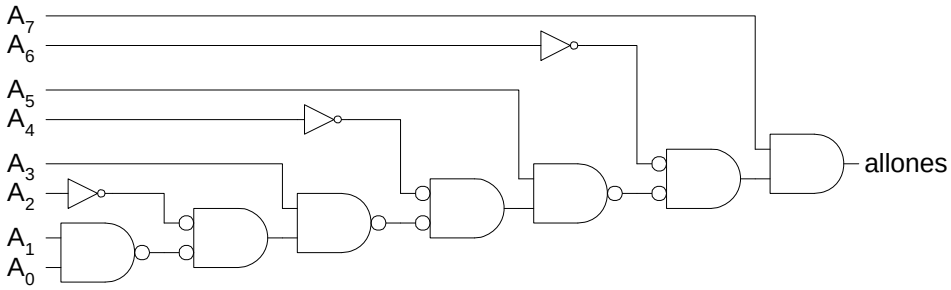
- 0's detector: A = 00…000
- 1's detector: A = 11…111
- Equality comparator: A = B
- Magnitude comparator: A < B

1's & 0's Detectors

- 1's detector: N-input AND gate
- 0's detector: NOTs + 1's detector (N-input NOR)

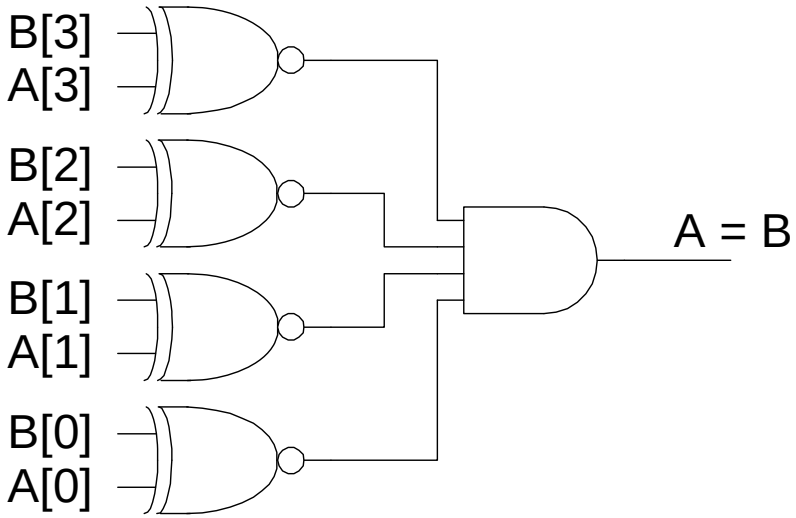






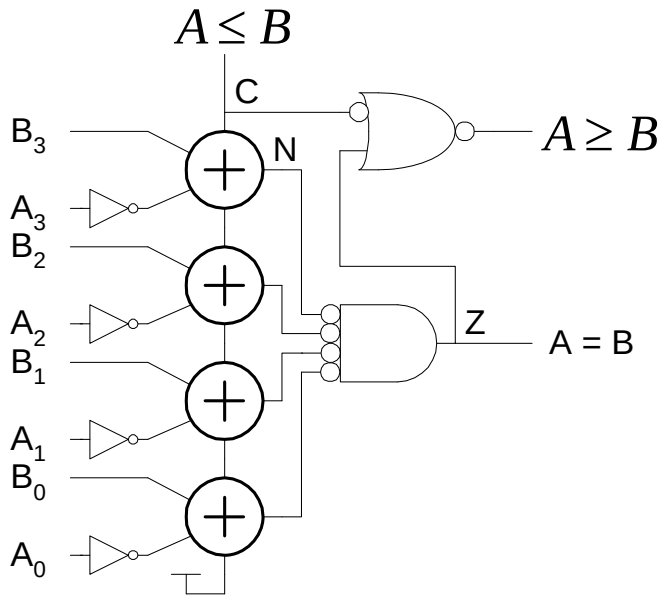
Equality Comparator

- Check if each bit is equal (XNOR, aka equality gate)
- 1's detect on bitwise equality



Magnitude Comparator

- Compute B-A and look at sign
- B-A = B + ~A + 1
- For unsigned numbers, carry out is sign bit



Signed vs. Unsigned

- For signed numbers, comparison is harder
  - C: carry out
  - Z: zero (all bits of A-B are 0)
  - N: negative (MSB of result)
  - V: overflow (inputs had different signs, output sign ≠ B)

Table 10.4 Magnitude comparison

Relation	Unsigned Comparison	Signed Comparison
$A = B$	Z	Z
$A \neq B$	$\overline{Z}$	$\overline{Z}$
$A < B$	$\overline{C + Z}$	$\overline{(N \oplus V) + Z}$
$A > B$	$\overline{C}$	$(N \oplus V)$
$A \leq B$	C	$(\overline{N \oplus V})$
$A \geq B$	$\overline{C} + Z$	$(N \oplus V) + Z$

Shifters

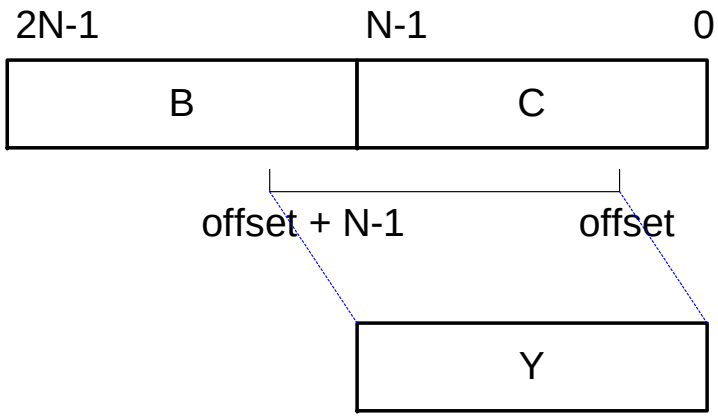
- Logical Shift:
  - Shifts number left or right and fills with 0's
    - 1011 LSR 1 = ____ 1011 LSL1 = ____
- Arithmetic Shift:
  - Shifts number left or right. Rt shift sign extends
    - 1011 ASR1 = ____ 1011 ASL1 = ____
- Rotate:
  - Shifts number left or right and fills with lost bits
    - 1011 ROR1 = ____ 1011 ROL1 = ____

Shifters

- Logical Shift:
  - Shifts number left or right and fills with 0's
    - 1011 LSR 1 = 0101 1011 LSL1 = 0110
- Arithmetic Shift:
  - Shifts number left or right. Rt shift sign extends
    - 1011 ASR1 = 1101 1011 ASL1 = 0110
- Rotate:
  - Shifts number left or right and fills with lost bits
    - 1011 ROR1 = 1101 1011 ROL1 = 0111

Funnel Shifter

- A funnel shifter can do all six types of shifts
- Selects N-bit field Y from 2N-bit input
  - Shift by k bits ($0 \leq k < N$)



Funnel Shifter Operation

Table 10.10 Funnel shifter operation

Shift Type	*B*	*C*	Offset
Logical Right	0…0	$A_{N-1}…A_0$	k
Logical Left	$A_{N-1}…A_0$	0…0	$N-k$
Arithmetic Right			
Arithmetic Left			
Rotate Right			
Rotate Left			

- Computing N-k requires an adder

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Arithmetic Left			
Rotate Right			
Rotate Left			

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Rotate Left			

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Rotate Left	$A_{N-1}…A_0$	$A_{N-1}…A_0$	$N-k$

- Computing N-k requires an adder

Simplified Funnel Shifter

- Optimize down to 2N-1 bit input

Table 10.11 Simplified funnel shifter

Shift Type	Z	Offset
Logical Right	$0..0, A_{N-1}...A_0$	k
Logical Left	$A_{N-1}...A_0, 0..0$	$\overline{k}$
Arithmetic Right		
Arithmetic Left		
Rotate Right		
Rotate Left		

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Arithmetic Left	$A_{N-1}...A_0, 0..0$	$\overline{k}$
Rotate Right	$A_{N-2}...A_0, A_{N-1}...A_0$	k
Rotate Left		

Simplified Funnel Shifter

- Optimize down to 2N-1 bit input

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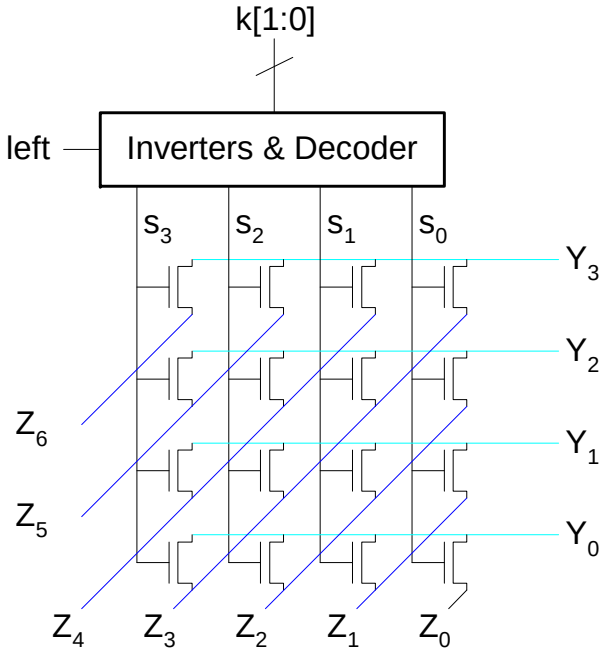
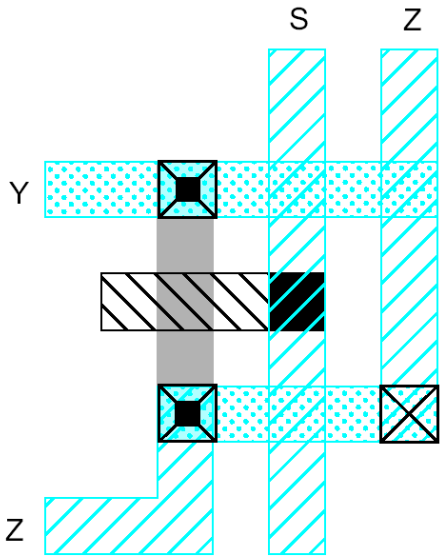
Rotate Left	$A_{N-1}...A_0, A_{N-1}..A_1$	$\overline{k}$

Funnel Shifter Design 1

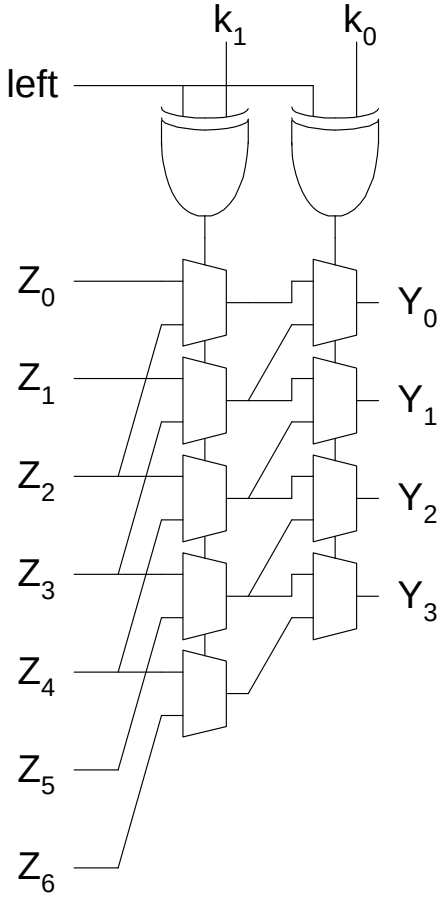
- N N-input multiplexers
  - Use 1-of-N hot select signals for shift amount
  - nMOS pass transistor design (V_t drops!)





Funnel Shifter Design 2

- Log N stages of 2-input muxes
  - No select decoding needed



Multi-input Adders

- Suppose we want to add k N-bit words
  - Ex: 0001 + 0111 + 1101 + 0010 = _____

Multi-input Adders

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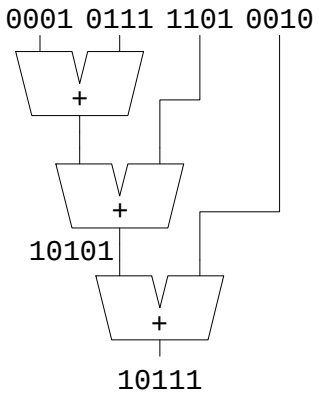
  - Ex: 0001 + 0111 + 1101 + 0010 = 10111

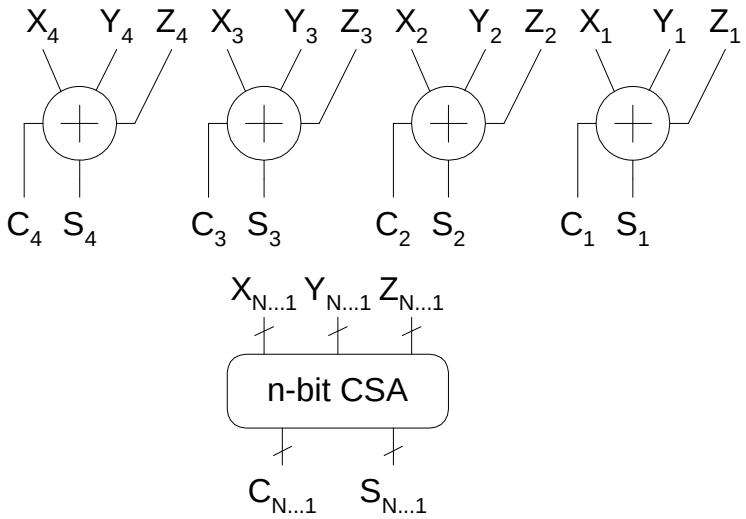
Multi-input Adders

- Suppose we want to add k N-bit words
  - Ex: 0001 + 0111 + 1101 + 0010 = 10111
- Straightforward solution: k-1 N-input CPAs
  - Large and slow



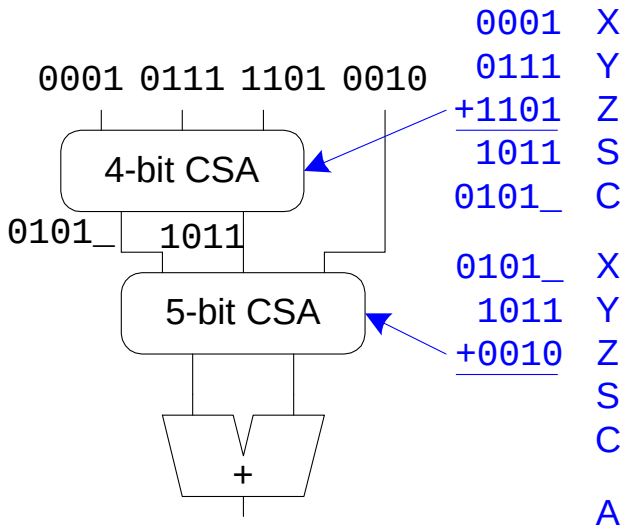
Carry Save Addition

- A full adder sums 3 inputs and produces 2 outputs
  - Carry output has twice *weight* of sum output
- N full adders in parallel are called *carry save adder*
  - Produce N sums and N carry outs



CSA Application

- Use k-2 stages of CSAs
  - Keep result in carry-save redundant form
- Final CPA computes actual result




```
 0001  X
 0111  Y
+1101  Z
 1011  S
0101_  C

0101_  X
 1011  Y
+0010  Z
       S
       C

       A
       B
       S
```

CSA Application

- Use k-2 stages of CSAs
  - Keep result in carry-save redundant form
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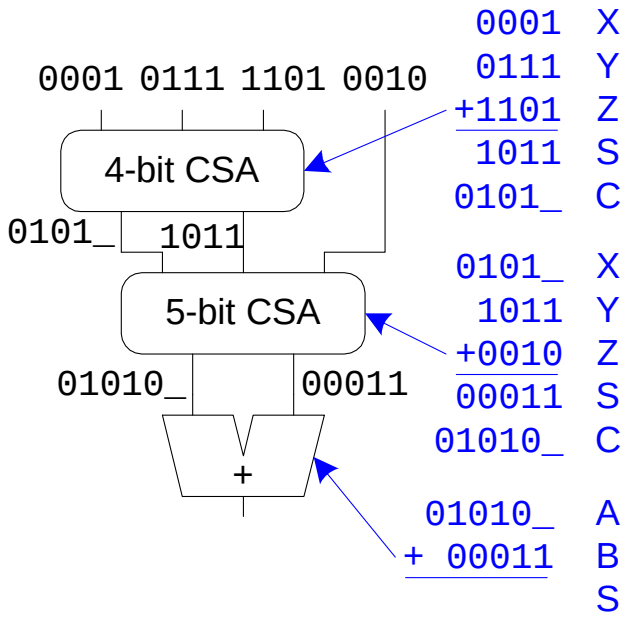

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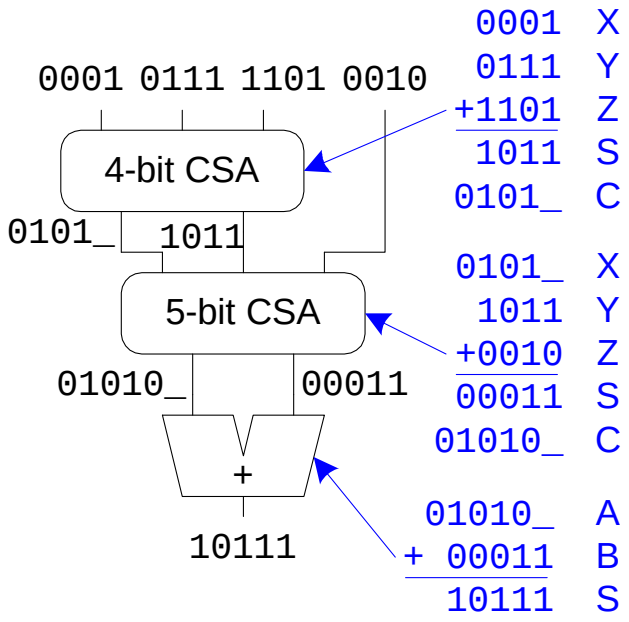


Multiplication

- Example:

$$\begin{array}{l} 1100 : 12_{10} \\ \underline{0101} : 5_{10} \\ \\ \hline \end{array}$$

Multiplication

- Example:

$$\begin{array}{rcl} 1100 & : & 12_{10} \\ \underline{0101} & : & 5_{10} \\ 1100 & & \end{array}$$

Multiplication

- Example:

$$
\begin{array}{rl}
1100 & : 12_{10} \\
\underline{0101} & : 5_{10} \\
1100 & \\
0000 & \\
\hline
\end{array}
$$

Multiplication

- Example:

$$
\begin{array}{rl}
1100 & : 12_{10} \\
\underline{0101} & : 5_{10} \\
1100 & \\
0000 & \\
1100 & \\
\hline
\end{array}
$$

Multiplication

- Example:

```
  1100 : 12_10
  0101 : 5_10
  ----
  1100
 0000
1100
0000
---------
```

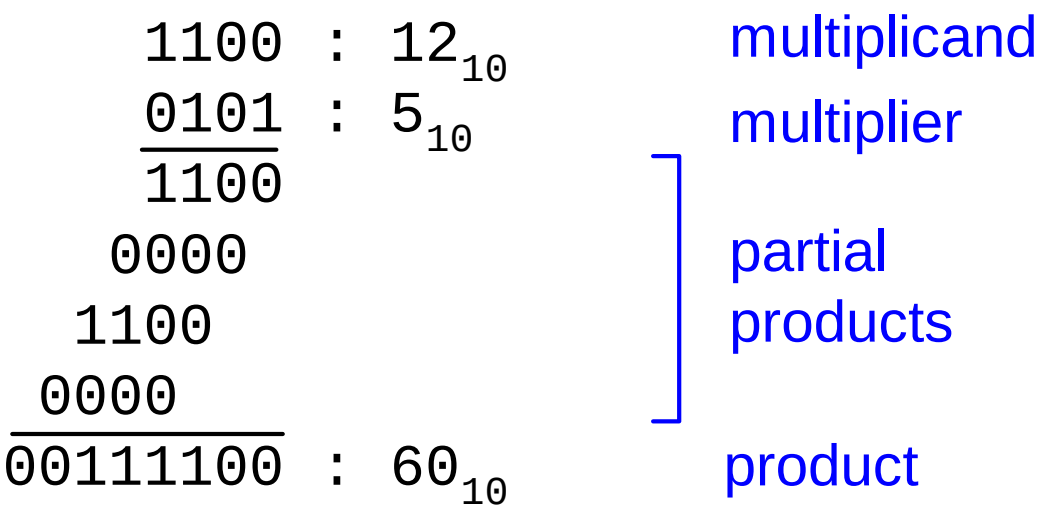
Multiplication

- Example:

```
   1100 : 12_10
   0101 : 5_10
   ----
   1100
  0000
 1100
0000
--------
00111100 : 60_10
```

Multiplication

- Example:

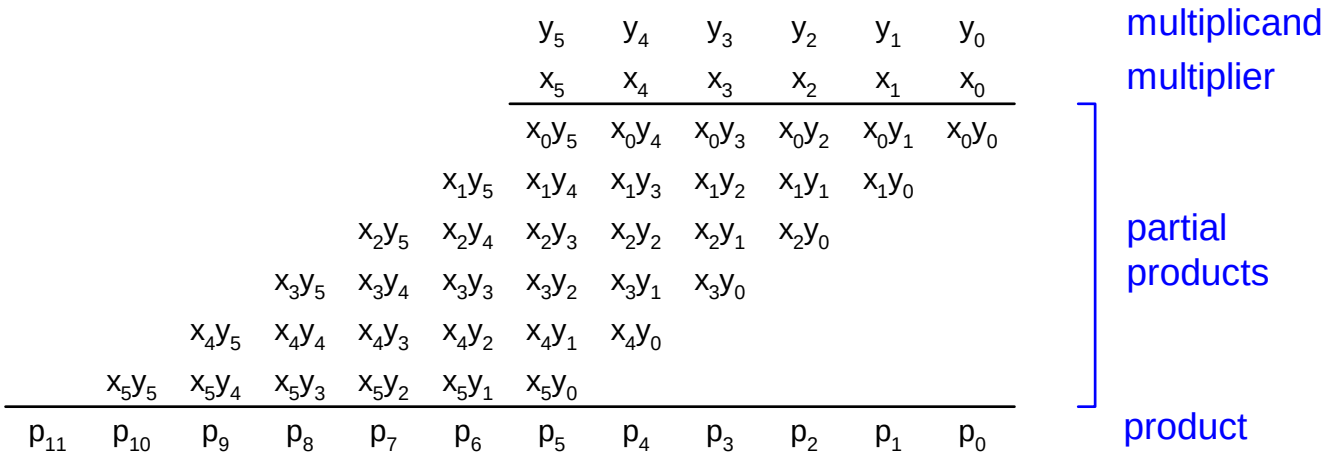


- M x N-bit multiplication
  - Produce N M-bit partial products
  - Sum these to produce M+N-bit product

General Form

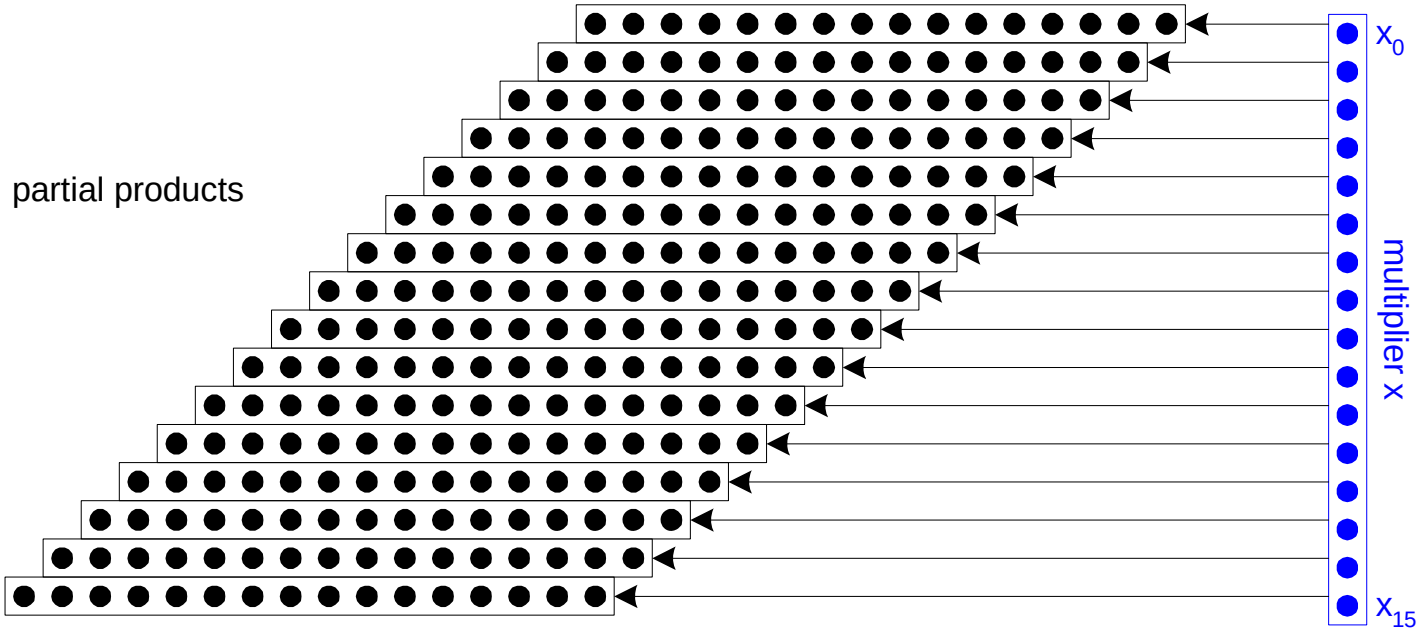
- Multiplicand: $Y = (y_{M-1}, y_{M-2}, \ldots, y_1, y_0)$
- Multiplier: $X = (x_{N-1}, x_{N-2}, \ldots, x_1, x_0)$
- Product:

$$P = \left(\sum_{j=0}^{M-1} y_j 2^j \right) \left(\sum_{i=0}^{N-1} x_i 2^i \right) = \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} x_i y_j 2^{i+j}$$

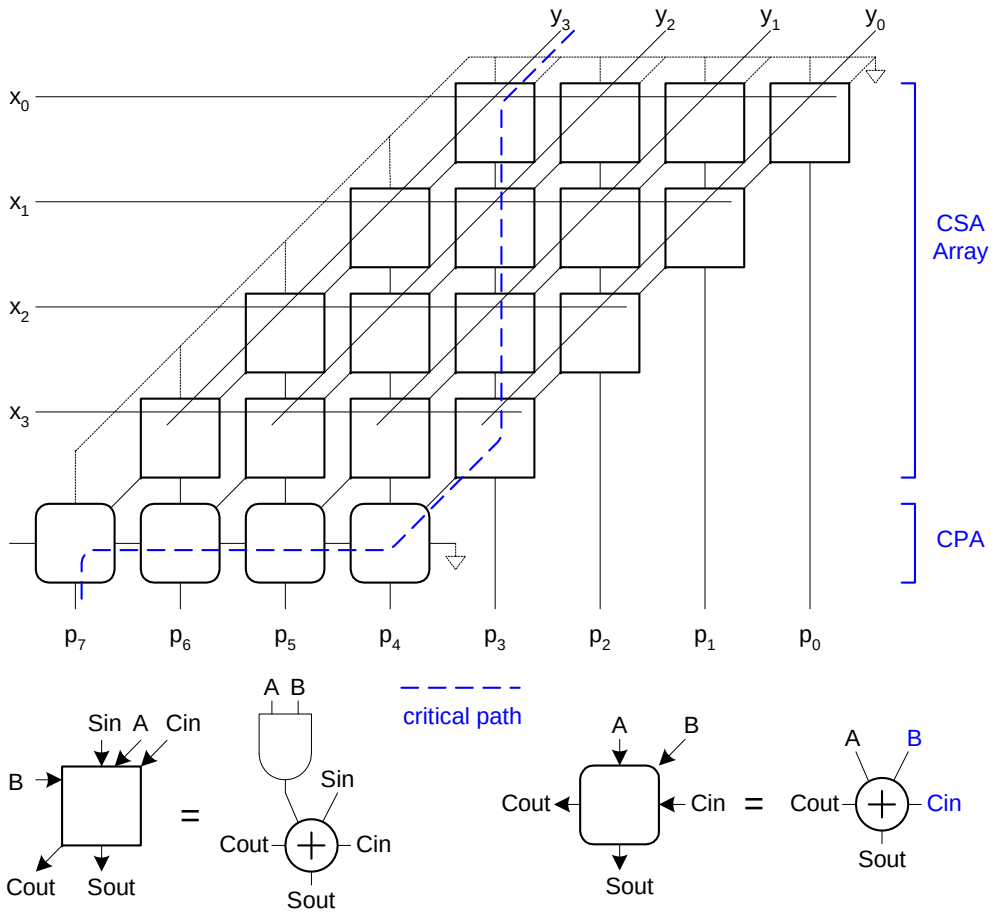


Dot Diagram

- Each dot represents a bit

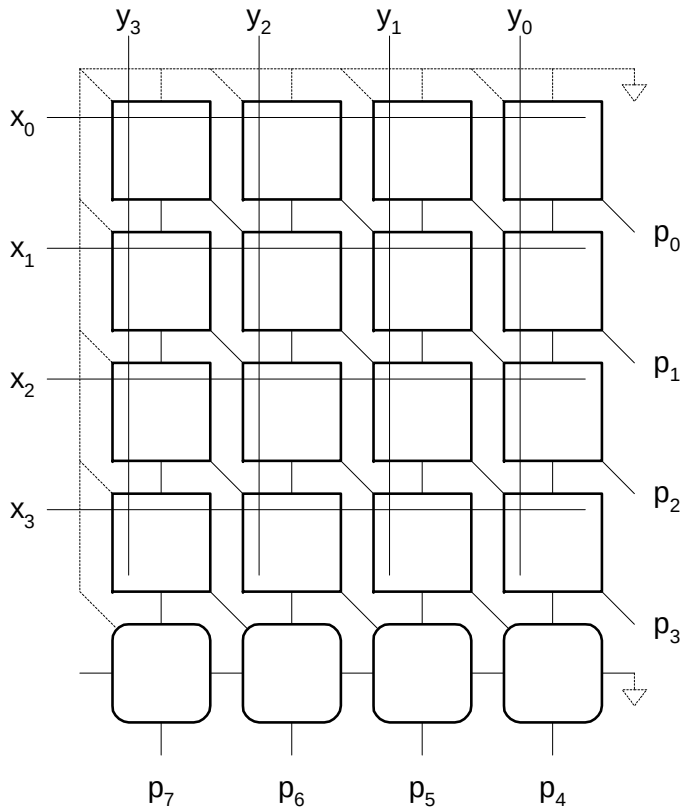


Array Multiplier



Rectangular Array

- Squash array to fit rectangular floorplan



Fewer Partial Products

- Array multiplier requires N partial products
- If we looked at groups of r bits, we could form N/r partial products.
  - Faster and smaller?
  - Called radix-2^r encoding
- Ex: r = 2: look at pairs of bits
  - Form partial products of 0, Y, 2Y, 3Y
  - First three are easy, but 3Y requires adder ☹

Booth Encoding

- Instead of 3Y, try –Y, then increment next partial product to add 4Y
- Similarly, for 2Y, try –2Y + 4Y in next partial product

Table 10.12 Radix-4 modified Booth encoding values

Inputs			Partial Product	Booth Selects		
x_{2i+1}	x_{2i}	x_{2i-1}	PP_i	X_i	$2X_i$	M_i
0	0	0	0			
0	0	1	Y			
0	1	0				
0	1	1				
1	0	0				
1	0	1				
1	1	0				
1	1	1				

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0	0	1	Y			
0	1	0	Y			
0	1	1				
1	0	0				
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0	0	1	Y			
0	1	0	Y			
0	1	1	$2Y$			
1	0	0				
1	0	1				
1	1	0				
1	1	1				

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0	0	1	Y			
0	1	0	Y			
0	1	1	$2Y$			
1	0	0	$-2Y$			
1	0	1				
1	1	0				
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0	0	1	Y			
0	1	0	Y			
0	1	1	$2Y$			
1	0	0	$-2Y$			
1	0	1	$-Y$			
1	1	0				
1	1	1				

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0	1	0	Y			
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1	0	0	$-2Y$			
1	0	1	$-Y$			
1	1	0	$-Y$			
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0	0	1	Y			
0	1	0	Y			
0	1	1	$2Y$			
1	0	0	$-2Y$			
1	0	1	$-Y$			
1	1	0	$-Y$			
1	1	1	$-0\ (=0)$			

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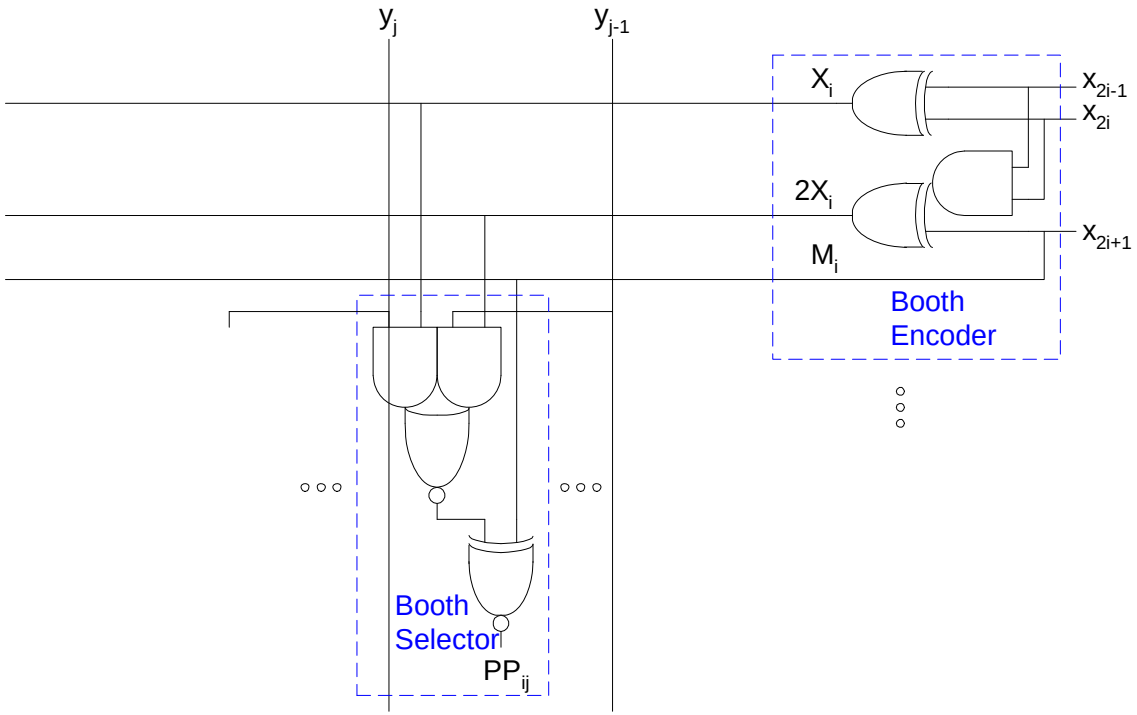
0	0	1	Y	1	0	0
0	1	0	Y	1	0	0
0	1	1	$2Y$	0	1	0
1	0	0	$-2Y$	0	1	1
1	0	1	$-Y$	1	0	1
1	1	0	$-Y$	1	0	1
1	1	1	$-0\ (=0)$	0	0	1

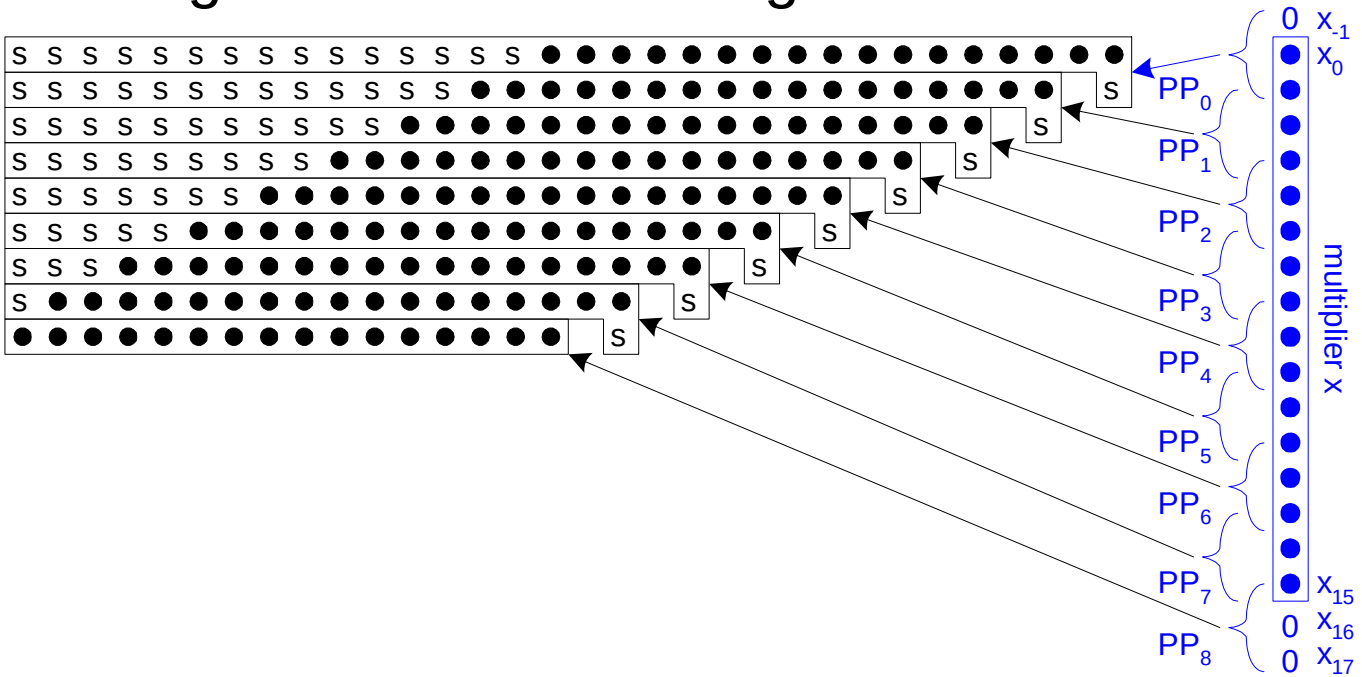
Booth Hardware

- Booth encoder generates control lines for each PP
  - Booth selectors choose PP bits



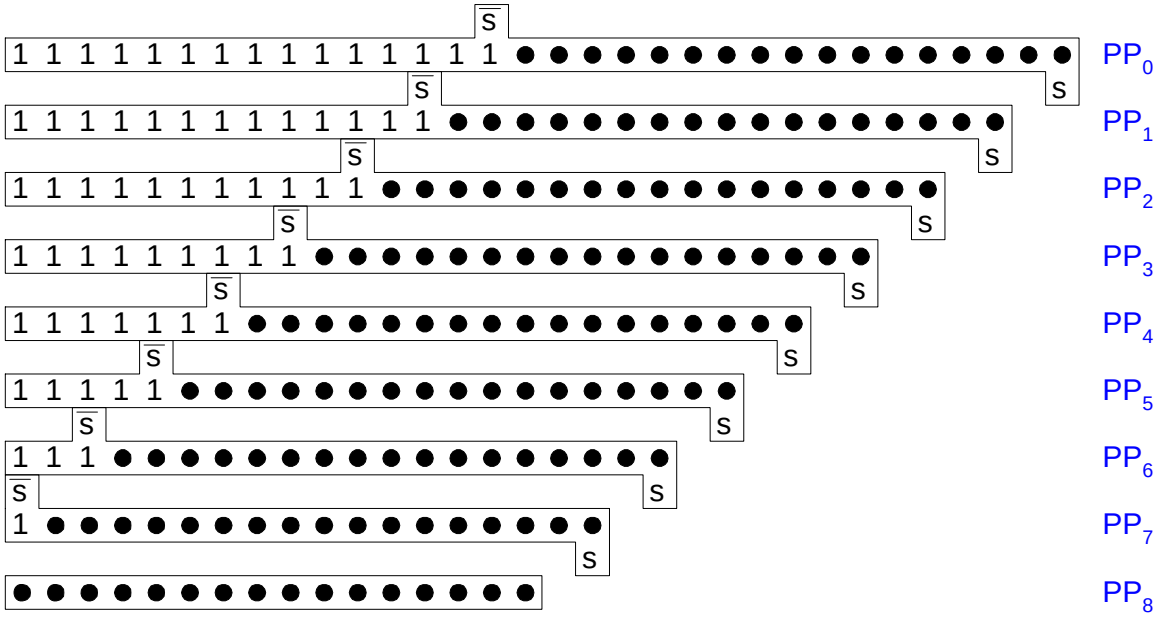
Sign Extension

- Partial products can be negative
  - Require sign extension, which is cumbersome
  - High fanout on most significant bit



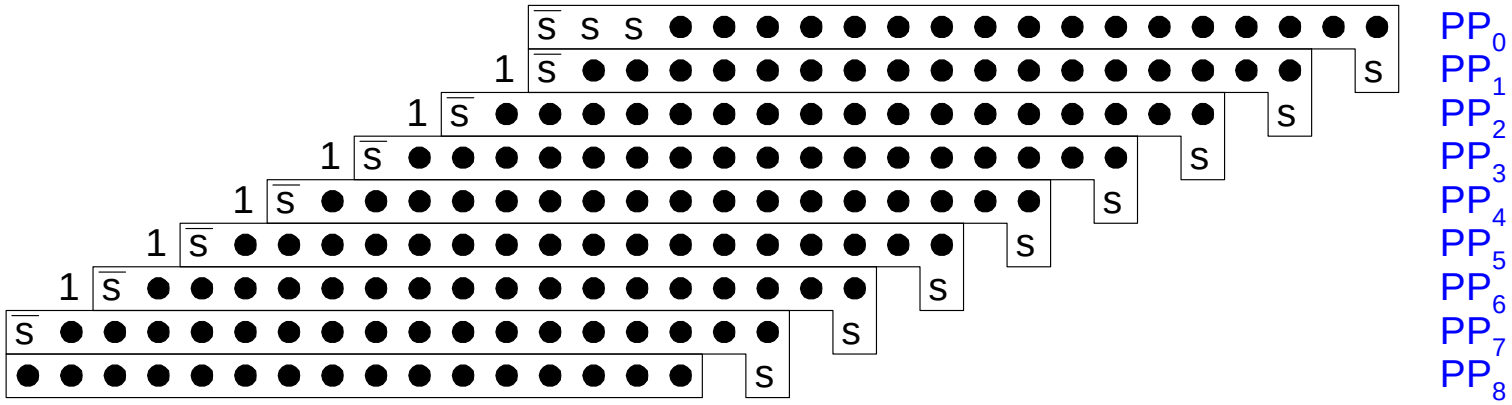
Simplified Sign Ext.

- Sign bits are either all 0's or all 1's
  - Note that all 0's is all 1's + 1 in proper column
  - Use this to reduce loading on MSB



Even Simpler Sign Ext.

- No need to add all the 1's in hardware
  - Precompute the answer!



Advanced Multiplication

- Signed vs. unsigned inputs
- Higher radix Booth encoding
- Array vs. tree CSA networks